

**Perform basic laboratory math skills = Apply and extend the properties of exponents to solve problems with rational exponents**

**Program Task:** Perform basic laboratory math skills.

**Program Associated Vocabulary:**

DECAY, GROWTH, HALF-LIFE, SHELF-LIFE

**Formulas and Procedures:**

In the Health Occupations field, as in the real world, things may double, but they also may decay. Numbers change, for example: the amount of horsepower an engine produces, the number of germs growing in a bottle, the depreciation value of a car, or the amount which aspirin decreases in our body after a few hours.

**Example:**

Right after ingesting aspirin, 20,000 PPM of a particular additive that reduces pain enters the blood stream. If the additive breaks down 25% an hour, how long before it reaches 1250 PPM and can longer be considered an effective analgesic?

**IV = Initial Value; NV = New Value; r = Rate; t = Time**

$$IV \times (1 - r)^t = NV$$

In this case we **SUBTRACT**

because the additive is decreasing.

$$20,000 \times (1 - .25)^t = 1250$$

$$20,000 \times (.75)^t = 1250$$

$$0.75^t = 1250/20,000$$

$$0.75^t = 0.0625$$

$$\log 0.75^t = \log 0.0625$$

$$t \times \log 0.75 = \log 0.0625$$

$$t = \log 0.0625 / \log 0.75$$

$$t = 9.6 \text{ hours}$$

**PA Core Standard:** CC.2.1.HS.F.1

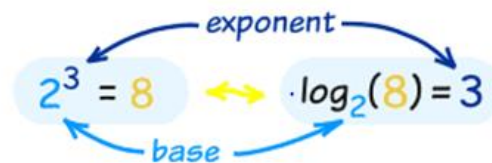
Description: Apply and extend the properties of exponents to solve problems with rational exponents.

**Math Associated Vocabulary:**

EXPONENTIAL FUNCTIONS, RAISING TO A POWER, EXPONENTS, LOGARITHMS, BASE

**Formulas and Procedures:**

Sometimes we have to use a mathematical process called a logarithm. The **logarithm** of a number is the **exponent** to which another fixed value, the **base**, must be raised to produce that number. For example, the logarithm of 8 to base 2 is 3, because 8 is 2 to the power 3:  $8 = 2 \times 2 \times 2 = 2^3$ .



**Example:**

James purchased a new car for \$18,000. Each year the car depreciates by 35% of its value the previous year. In how many years will it be worth \$600?

**Solution:**

**IV = Initial Value; NV = New Value; r = Rate; t = Time**

$$NV = IV(1 - r)^t$$

$$600 = 18,000(1 - 0.35)^t$$

$$\frac{600}{18,000} = (1 - 0.35)^t$$

$$0.033 = 0.65^t$$

$$\log 0.033 = \log 0.65^t$$

$$\log 0.033 = t \times \log 0.65$$

$$\frac{\log .033}{\log 0.65} = t$$

$$7.89 \text{ years} = t$$

### Instructor's Script - Comparing and Contrasting

The growth rate ( $r$ ) is the fractional amount added or removed within a given time period.

- If the amount is increasing, ( $r$ ) will be added to 1; if decreasing, ( $r$ ) will be subtracted from 1.
- Many times, the growth rate is given as a percentage and must be converted to a decimal first. ( $70\% = 70 \div 100 = 0.70$ )
- For half-life problems, use a growth rate of  $-50\%$  or  $-0.50$  ( $1 + r = 1 - 0.50 = 0.50$ )
- For doubling problems, use a growth rate of  $100\%$  or  $+1.00$  ( $1 + r = 1 + 1 = 2$ )

### Properties of Logarithms

- 1)  $\log_b(mn) = \log_b(m) + \log_b(n)$
- 2)  $\log_b(m/n) = \log_b(m) - \log_b(n)$
- 3)  $\log_b(m^n) = n \times \log_b(m)$

### Common Mistakes Made By Students

Not performing the order of operations correctly: Parentheses ( $1 + r$ ), Exponent ( $n$ ), Multiplication ( $A_0$ ).

- Setting  $r$  to the amount of material remaining after a change instead of the growth or decay change (“ $1+r$ ” is the amount remaining after growth).

Setting the sign of  $r$  incorrectly (“ $1 + r$ ” should be greater than 1.0 if growing and less than 1.0 if decaying).

### CTE Instructor's Extended Discussion

Technical tasks are usually not presented using this model. Therefore, it is important for technical instructors to demonstrate to students how these math concepts link to and are relevant in their technical training, and that technical teachers present the math concepts in a way which shows a relationship to the math which CTE students use in their academic school settings.

To become a better instructor, make it your business to reach the comfort level necessary for teaching the math concepts and formulas that make Health Occupations the profitable and satisfying career that we all know it can be.

Problems	Career and Technical Math Concepts	Solutions
<p>1. A particular reagent in solution contains 75% its original formulation. The solution has a half-life of 10 years. How old is the reagent in solution?</p> <p style="text-align: center;">Half-life formula: <math>NV = \left(\frac{1}{2}\right)^{T/\text{half}}</math></p>		
<p>2. There are 800,000 bacteria growing on a culture dish. After administering antibiotics, the bacteria are disappearing at a rate of 12% a day. How many days before there are only 10,000 bacteria remaining and the infection considered cured?</p>		
<p>3. A new drug supplement promises to increase a particular hormone by 5% a month, which should hopefully eliminate a patient's symptom. Currently the level is 140 ppm, but the body requires 400 ppm. How long will it take to meet required levels of the chemical?</p>		
Problems	Related, Generic Math Concepts	Solutions
<p>4. Brad created a chart that shows the population of a town will increase to 98,321 people from a current population of 12,566 people. The rate of increase is an annual increase of 5.25%. Brad forgot to include the number of years this increase will take. How many years was it?</p>		
<p>5. In 1957, Chevrolet built 320,000 Belair cars. These classics are now disappearing at a rate of 20% a year. In 2007 there were still 12,500 of these vehicles driving around. How many years before there are only 100 left on the road?</p>		
<p>6. A city currently has a population of 15,600. The number of people living in the city doubles every 10 years. How many people will be living in the city in 18 years, 24 years and 35 years?</p>		
Problems	PA Core Math Look	Solutions
<p>7. <math>14^x = 86</math> Solve for x.</p>		
<p>8. <math>\log(2x) = 4</math> Solve for x.</p>		
<p>9. <math>\log(2x) = 2</math> Solve for x.</p>		

Problems	Career and Technical Math Concepts	Solutions
1. A particular reagent in solution contains 75% its original formulation. The solution has a half-life of 10 years. How old is the reagent in solution?  Half-life formula: $NV = \left(\frac{1}{2}\right)^{T/\text{half}}$	$= \log_5 5^{T/10} = .75 = \frac{t}{10} \log_5 5 = \log_5 75$  Multiple both sides by 10 to cancel $\frac{t}{10}$  $\frac{10 \log_5 75}{\log_5 50} = 4.15 \text{ years}$	
2. There are 800,000 bacteria growing on a culture dish. After administering antibiotics the bacteria are disappearing at a rate of 12% a day. How many days before there are only 10,000 bacteria remaining and the infection considered cured?	$Q = Q_0(1-r)^t = 800,000(1-.12)^t = 10,000 = 800,000(.88)^t = 10,000$ Divide both sides by 800,000  $= \frac{\log .0125}{\log .88} = 31.4 \text{ days}$	
3. A new drug supplement promises to increase a particular hormone by 5% a month, which should hopefully eliminate a patient's symptoms. Currently the level is 140 ppm, but the body requires 400 ppm. How long will it take to meet required levels of the chemical?	$IV \times (1+r)^t = NV \quad IV \times (1.05)^t = 400 \text{ ppm}$ $140 \text{ ppm} \times 1.05^t = 400 \text{ ppm}$ Divide both sides by 140  $\frac{147}{140} \frac{400}{140} = \frac{\log 2.86}{\log 1.05} = 21.5 \text{ months}$	
Problems	Related, Generic Math Concepts	Solutions
4. Brad created a chart that shows the population of a town will increase to 98,321 people from a current population of 12,566 people. The rate of increase is an annual increase of 5.25%. Brad forgot to include the number of years this increase will take. How many years was it?	$98,321 = 12,566(1 + 0.0525)^t$ $98,321/12,566 = (1.0525)^t$ $7.824 = 1.0525^t$ $\log 7.824 = \log 1.0525^t$ $\log 7.824 = t \times \log 1.0525$ $\log 7.824 / \log 1.0525 = t$ therefore $t = 40.2 \text{ years}$	
5. In 1957, Chevrolet built 320,000 Belair cars. These classics are now disappearing at a rate of 20% a year. In 2007 there were still 12,500 of these vehicles driving around. How many years before there are only 100 left on the road?	$Q = Q_0(1-r)^t = 12,500(1-.20)^t = 100 = 12,500(.80)^t = 100$ Divide both sides by 12,500  $= \frac{\log .008}{\log .80} = 21.64 \text{ years}$	
6. A city currently has a population of 15,600. The number of people living in the city doubles every 10 years. How many people will be living in the city in 18 years? 24 years? And 35 years?	$NV = IV(2)^{(18/10)} \quad NV = 54,322$ $NV = IV(2)^{(24/10)} \quad NV = 82,337$ $NV = IV(2)^{(35/10)} \quad NV = 176,493$	
Problems	PA Core Math Look	Solutions
7. $14^x = 86$ Solve for x.		$\frac{\log 86}{\log 14} = 1.688 \quad x = 1.688$ Check your work: $14^{1.688} = 86.032$
8. $\log(2x) = 4$ Solve for x.		$\log 2x = 4, 2x = 10^4, 2x = 10,000$ $x = 5000$ Check your work: $\log 2 \times 5000 = 4$
9. $\log(2x) = 2$ Solve for x.		$\log(2x) = 2, 2x = 10^2, 2x = 100, \div \text{ both sides by } 2, x = 50$ Check your work: $\log (2 \times 50) = 2$